Robust Tracking Control of Kinematically Redundant Robot Manipulators Subject to Multiple Self-Motion Criteria†

Erkan Zergeroğlu, Hüsnü Türker Şahin, Ufuk Özbay and Ümit Ali Tektaş
Department of Computer Engineering, Gebze Institute of Technology,
PK. 141, 41400 Gebze/Kocaeli-Turkey.
E-mail: {ezerger,htsahin,ufuk,tektasu}@bilmuh.gyte.edu.tr

***Abstract*— In this study, we consider a model based robust control scheme for kinematically redundant robot manipulators that also enables the use of self motion of the manipulator to perform multiple sub-tasks (like, maintaining manipulability, avoidance of mechanical joint limits and obstacle avoidance). The controller proposed ensures uniformly ultimately bounded end-effector and sub-task tracking despite the parametric uncertainty associated with the dynamic model. The controller design has been based on a Lyapunov type approach. Simulation results performed on a 3 link planar robot arm are presented to demonstrate the capabilities and the performance of the controller.**

I. INTRODUCTION

It is well known that to achieve better performance, it is imperative to incorporate the nonlinear robot dynamics into the controller design. However robot dynamics not only exhibits high nonlinearities but also is hard to model accurately. Additionally, external disturbances are inevitable under real circumstances. Therefore robot controllers have to be robust to the dynamical parametric uncertainties as well as external additive disturbances. The control design problem becomes more sophisticated when the desired motion of a robot manipulator is defined at the end effector frame level, referred as *"operational space"*. For non-redundant manipulators there is a unique relation between the joint variables and the end effector position/orientation. However this is not the case for kinematically redundant robot manipulators.

Kinematically redundant manipulators have more degrees of freedom (DOF) than is required to perform a task in the operational space; hence, these extra degrees of freedom allow the robot manipulator to perform more dextrous manipulation and/or provide the robot manipulator system with increased flexibility for the execution of sophisticated tasks. Since the dimension (*i.e.*, n) of the link position variables is greater than the dimension (*i.e.*, m) of the operational space variables, the null space of Jacobian matrix has a minimum dimension of $n - m$. That is, any link velocity in the null space of the manipulator Jacobian will not effect the operational space velocity. This motion of the links is referred to as *self-motion* since it is not observed in the operational space. As stated in [1], [2], and [3], there are generally an infinite number of solutions for the inverse kinematics of a redundant manipulator. Thus, given a desired end effector trajectory, it is difficult to select a reasonable joint space trajectory that satisfies both control constraints (*i.e.*, stability and boundedness of all signals) and mechanical constraints (*i.e.*, singularities and joint limit avoidance). Therefore an efficient controller for kinematically redundant robot manipulators should incorporate the robot dynamics while being robust to parametric uncertainties associated with the dynamics and external disturbances, and achieve accurate end effector tracking while letting the self motion of the manipulator available for performance enhancement.

Due to the challenging nature of the forementioned control design problem, many researches attacked it proposing different types of controllers. To name a few; in [4], Khatib proposed a control scheme based on the dynamic model of a manipulator in Cartesian space and extended this result for redundant manipulators by using the pseudo-inverse of the Jacobian matrix. In [5], Seraji proposed the configuration control approach in which the end-effector motion in task-space is augmented by any $n - m$ dimensional additional tasks, such as optimization of kinematic and dynamic objectives or posture control. In [6], Hsu *et.al.* proposed a dynamic feedback linearizing control law that guarantees end-effector tracking and also provides control of redundant link velocities. In [7], Colbaugh *et.al.* proposed a robust adaptive controller that ensures globally ultimately bounded Cartesian tracking provided that no external disturbances are present in the robot dynamics and some sufficient conditions on control gains are satisfied. In [8], Peng *et.al.* proposed a compliant motion control for kinematically redundant manipulators using an extended task-space formulation. In [9], Oh *et.al.* proposed a disturbance observer based robust controller that controls both the motion of the end effector and the null space motion of the redundant manipulator. The controller proposed used an extended operational space formulation to express both operational space and null space dynamics. Recently in [10], Zergeroglu *et.al.* presented a model based controller that achieves exponential end-effector and sub-task tracking. The extensions for adaptive and model based output feedback type of controllers were also presented, however the controllers proposed either required the exact knowledge of the dynamics (for the full state and output feedback controllers) or the uncertain robot dynamics to be linearly parametrizable (for adaptive controller extension) and did not

† This work is mainly supported by the Scientific and Technological Research Council of Turkey (TUBITAK) Project No: 104 E061.

take the external disturbances into account.

As can be seen from an examination of the cited papers in the above paragraph, the previous research on control of kinematically redundant manipulators is mainly concentrated on two major approaches. The first approach is the extended or augmented operational space formulation. In this approach, the dimension of the operational space is extended by incorporating as many additional constraints as the degree of the redundancy, and hence, the resulting system becomes non-redundant. Unfortunately, this approach usually introduces additional algorithmic singularities related to rank of the so-called extended Jacobian matrix, and hence, can cause the control input to become unbounded even though the manipulator is not in a singular position. The second approach is the generalized/pseudo-inverse based control formulations that use the pseudo-inverse of the manipulator Jacobian in the control formulation. While this approach can not guarantee that the control remains bounded when the manipulator is near singularities associated with the standard Jacobian matrix, it does not introduce additional singularity issues.

In this paper, we consider the design of a robust controller for kinematically redundant robot manipulators using a generalized/pseudo-inverse based formulation. Specifically we have developed a robust controller that achieves uniformly ultimately bounded end-effector and sub-task tracking despite the parametric uncertainties associated with the dynamics and additive external disturbances. When compared with the previously proposed controllers; with respect to [6] the controller proposed is robust to the parametric uncertainties in the robot dynamics. With respect to [9], the proposed controller does not introduce additional singularity issues due to the extended Jacobian formulation and with respect to [10] it is capable of compensating a larger class of uncertainties. Moreover, in this work we have further investigated the use and manipulation of multiple sub-task criteria to analyze their effects on the performance of robots.

The rest of the paper is organized as follows. Section II presents the kinematic and dynamic properties of redundant robot manipulators. Section III states the control objective and details the error system development. The use of multiple sub-task criteria for possible performance enhancement is presented in Section IV. Simulation results are given in Section V. Finally Section VI contains concluding remarks.

II. ROBOT MODEL

A. Kinematic Model

The end-effector position and orientation in the operation space, denoted by $x(t) \in \mathbb{R}^m$, is defined as a function of joint position vector as [11]

$$x = f(q) = \begin{bmatrix} p(q) \\ \phi(q) \end{bmatrix} \tag{1}$$

where $f(q) \in \mathbb{R}^m$, $m \in \mathbb{Z}$ is the forward kinematic calculations, $q(t) \in \mathbb{R}^n$ denote the link position vector of an n-link manipulator $p(q) \in \mathbb{R}^l$ and $\phi(q) \in \mathbb{R}^{(m-l)}$ are the vectors representing the end-effector position, and orientation respectively and $l \in \mathbb{Z}$ is the size of the operational space.

Based on (1), the differential relationships between the end-effector position and the link position variables, is obtained as follows

$$\begin{aligned} \dot{x} &= J(q)\,\dot{q} \\ \ddot{x} &= \dot{J}(q)\dot{q} + J(q)\ddot{q} \end{aligned} \tag{2}$$

where $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^n$ denote the link velocity and acceleration vectors, respectively and $J(q) \triangleq \partial f(q)/\partial q \in \mathbb{R}^{m\times n}$, is the Jacobian matrix of the manipulator. Note that for kinematically redundant manipulators the joint velocities may also be represented using (2) as follows

$$\begin{aligned} \dot{q} &= J^+\dot{x} + (I_n - J^+J)\,g \\ &= J^+\dot{x} + k\,(I_n - J^+J)\,[\nabla H(q)] \end{aligned} \tag{3}$$

where $J^+(q) \in \mathbb{R}^{n\times m}$ is the pseudo-inverse of the manipulator Jacobian and is defined in the following form [12]

$$J^+ = J^T\left(JJ^T\right)^{-1} \text{ such that } \quad JJ^+ = I_m. \tag{4}$$

In (3) $I_n \in \mathbb{R}^{n\times n}$ denotes the $n \times n$ identity matrix, $(I_n - J^+J)$ is the null space projection matrix. $J^+\dot{x}$ is the minimum norm joint velocity solution, $(I_n - J^+J)\,g$ is a homogenous solution of (3) in the null space of J orthogonal to $J^+\dot{x}$ and $g(t) \in \mathbb{R}^n$ is an auxiliary joint velocity vector which can be constructed to improve the performance of the manipulator according to the sub-task control objective (*e.g.*, mechanical limit avoidance, or obstacle avoidance). This possible performance enhancement is achieved by optimizing a proper performance criterion function, $H(q) \in \mathbb{R}$, where $\nabla H(q)$ is the gradient of $H(q)$ and k is a real valued scaler. The pseudo-inverse defined by (4) satisfies the Moore-Penrose Conditions [1], [13] given below,

$$\begin{aligned} &JJ^+J = J, &&J^+J\,J^+ = J^+, \\ &(J^+J)^T = J^+J, &&(JJ^+)^T = JJ^+ \end{aligned} \tag{5}$$

and the null space matrix $(I_n - J^+J)$ satisfies the following properties

$$\begin{aligned} &(I_n - J^+J)(I_n - J^+J) = I_n - J^+J, &&J(I_n - J^+J) = 0, \\ &(I_n - J^+J)^T = (I_n - J^+J), &&(I_n - J^+J)J^+ = 0. \end{aligned} \tag{6}$$

B. Dynamic Model

The dynamic model for an n-link, revolute, direct drive robot manipulator is assumed to be in the following form [11]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \xi_d = \tau \tag{7}$$

where $M(q) \in \mathbb{R}^{n\times n}$ represents the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ is the gravity vector, $F(\dot{q}) \in \mathbb{R}^n$ represents the friction effects, $\xi_d \in \mathbb{R}^n$ is a vector containing the unknown but bounded, additive disturbance effects and $\tau(t) \in \mathbb{R}^n$ is the torque input vector.

The left-hand side of (7) can be separated as follows

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+F(\dot{q})+\xi_d = W(q,\dot{q},\ddot{q})\,\theta+\varsigma \tag{8}$$

where $W(q,\dot{q},\ddot{q})\,\theta$ contains the linearly parametrizable (LP) part of (7) and $\varsigma \in \mathbb{R}^n$ contains the rest of the terms that are not LP. In equation (8) $\theta \in \mathbb{R}^r$ contains some of the constant system parameters (i.e. mass of the links, center of mass of a link, etc.), and the regression matrix $W(\cdot) \in \mathbb{R}^{n\times r}$

contains known functions dependent on the signals $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)$ (it is assumed that if the arguments of $W(\cdot)$ are bounded then $W(\cdot)$ is bounded).

During the control development, we will make the commonly used assumption that the minimum singular value of the manipulator Jacobian, denoted by σ_m is greater than a known small positive constant $\delta > 0$, such that $\max\{\|J^+(q)\|\}$ is known a prior and all kinematic singularities are always avoided. We also note that since we are only concerned with revolute robot manipulators, we know that kinematic and dynamic terms denoted by $M(q)$, $C(q,\dot{q})$, $G(q)$, $J(q)$, and $J^+(q)$ are bounded for all possible $q(t)$ (*i.e.*, these kinematic and dynamic terms only depend on $q(t)$ as arguments of trigonometric functions).

III. CONTROL AND ERROR SYSTEM FORMULATION

Our control objective is to design the control torque input signal $\tau(t)$ such that the robots end effector in the operation space can follow a desired end effector position and orientation signal as closely as possible. Additional to this main objective the designed control signal should also enable the redundancy of the manipulator to execute subtasks defined by at least one motion optimization measure (such as joint limit avoidance and/or obstacle avoidance). To achieve this, we will refer the task space tracking as our "main" objective and enabling the use of manipulators redundancy as our secondary or "sub-task" objective.

To quantify our main control objective, the operational space tracking error $e(t) \in \mathbb{R}^m$, is defined as follows

$$e = x_d - x \tag{9}$$

where $x_d(t) \in \mathbb{R}^m$ denotes the desired operational space trajectory where it is assumed that $x_d(t)$, $\dot{x}_d(t)$, and $\ddot{x}_d(t)$ are all bounded functions of time. Similar to [6], we define the sub-task tracking error, denoted by $e_N(t) \in \mathbb{R}^n$, as follows

$$e_N = (I_n - J^+J)(g - \dot{q}) \tag{10}$$

where g was defined in (3). To provide motivation for the definition of the sub-task control objective given by (10), we take the time derivative of (9), and then substitute (2) for $\dot{x}(t)$ to obtain

$$\dot{e} = \dot{x}_d + \alpha e - \alpha e - J\dot{q} \tag{11}$$

where the term αe has been added and subtracted to right-hand side of (11) to facilitate the control formulation, and $\alpha \in \mathbb{R}^{m\times m}$ denotes a diagonal, positive definite gain matrix. Using the properties of the pseudo-inverse of the manipulator Jacobian defined in (5), we can rewrite (11) in the following advantageous form

$$\dot{e} = -\alpha e + J\left(J^+(\dot{x}_d + \alpha e) + (I_n - J^+J)g - \dot{q}\right) \tag{12}$$

Based on the structure of (12) and the subsequent analysis, we define a filtered tracking error signal, denoted by $r(t) \in \mathbb{R}^n$, as follows

$$r \triangleq J^+(\dot{x}_d + \alpha e) + (I_n - J^+J)g - \dot{q}; \tag{13}$$

hence, the closed-loop operational space position tracking error system can now be written into the final form

$$\dot{e} = -\alpha e + Jr. \tag{14}$$

Based on the structure of (14), we are motivated to regulate $r(t)$ in order to compensate $e(t)$; hence, we must calculate the open-loop dynamics for $r(t)$. To this end, we take the time derivative of (13), pre-multiply by the inertia matrix $M(q)$, and then substitute (7) to yield the following advantageous form for the open loop dynamics

$$M\dot{r} = \quad -Cr + \omega_r - \tau \tag{15}$$

where the function $\omega_r(t)$ is defined explicitly as follows

$$\begin{aligned}\omega_r \triangleq \quad & M(q)\frac{d}{dt}\{J^+(\dot{x}_d + \alpha e) + (I_n - J^+J)g\} \\ & + C(q,\dot{q})\{J^+(\dot{x}_d + \alpha e) + (I_n - J^+J)g\} \\ & + G(q) + F(\dot{q}) + \xi_d.\end{aligned} \tag{16}$$

Note that applying (8), $\omega_r(t)$ can also be separated into two parts in the following way

$$\omega_r = Y\theta + \xi_r \tag{17}$$

where $Y\theta$ is defined as the regression matrix/parameter vector formulation with $Y(\ddot{x}_d, \dot{x}_d, x, q, \dot{q}, \dot{g}, g) \in \mathbb{R}^{n\times r}$ denoting the regression matrix, $\theta \in \mathbb{R}^r$ contains the constant system parameters (*e.g.*, mass, inertia, friction coefficients) and $\xi_r \in \mathbb{R}^n$ contains the rest of the parameters of ω_r that are not linearly parametrizable.

In the following section of the paper, we will use the structure of (14) to design the control input to ensure that the operational space error and the filtered tracking error defined by (9) and (13), respectively, are both regulated uniformly inside an ultimate, adjustable bound. To illustrate how the regulation of the filtered tracking error also ensures regulation of the sub-task tracking error defined by (10), we pre-multiply (13) by $(I_n - J^+J)$ and apply the properties given in (6) to obtain

$$e_N = (I_n - J^+J)r \tag{18}$$

where (10) has been utilized. From (18), it is clear that when $r(t)$ is regulated then $e_N(t)$ is also regulated, and hence, the sub-task control is also achieved.

A. Control Design and Analysis

Based on the above error system development and the subsequent stability analysis, we design the control torque input $\tau(t)$ as follows

$$\tau = Y\hat{\theta} + Kr + \upsilon_R + J^T e \tag{19}$$

where $\hat{\theta} \in \mathbb{R}^r$denotes the constant best guess estimates of the parameter vector, θ, is defined in (17), $K \in \mathbb{R}^{n\times n}$ is a constant, positive definite, diagonal gain matrix and $\upsilon_R \in \mathbb{R}^n$ is an auxiliary robust control term defined explicitly as

$$\upsilon_R = \frac{r\rho^2}{\|r\|\rho + \varepsilon} \tag{20}$$

with the positive bounding function $\rho(\cdot)$ designed to satisfy

$$\rho \geq \left\|Y\tilde{\theta}\right\| + \|\xi_r\| \tag{21}$$

and ε being a positive small scaler constant. In addition $\tilde{\theta} \in \mathbb{R}^r$ given in (21) is the parameter estimation error defined as the difference between the actual and the estimated parameters as

$$\tilde{\theta} \triangleq \theta - \hat{\theta} \tag{22}$$

In the subsequent analysis the following fact will also be utilized

$$\left\|\tilde{\theta}\right\| \leq \zeta_\theta \tag{23}$$

where $\zeta_\theta \in \mathbb{R}$ denotes a known positive bounding constant. After substituting (19) and (17) into (15), the closed-loop error system for $r(t)$ can be written in the following form

$$M\dot{r} = -Cr + Y\tilde{\theta} + \xi_r - J^T e - \upsilon_R - Kr. \tag{24}$$

We now state the following result.

Theorem 1: The robust control law described by (19) guarantees uniformly ultimately bounded (UUB) task-space end effector position and sub-task tracking in the sense

$$\|e(t)\| \leq \sqrt{\tfrac{a}{b}\|z(0)\|^2 \exp(-\gamma t) + \tfrac{2\varepsilon}{b\gamma}(1-\exp(-\gamma t))},$$
$$\|e_N(t)\| \leq \beta\sqrt{\tfrac{a}{b}\|z(0)\|^2 \exp(-\gamma t) + \tfrac{2\varepsilon}{b\gamma}(1-\exp(-\gamma t))} \tag{25}$$

where $z \triangleq \begin{bmatrix} r^T & e^T \end{bmatrix}^T$ *and auxiliary variables* a, b, γ, β *are explicitly defined as follows*

$$a = \max(m_2, 1), \quad b = \min(m_1, 1),$$
$$\gamma = \frac{2\min(\alpha, \lambda_{\min}(K))}{\max(m_2, 1)}, \quad \beta = \|I_n - J^+ J\|_{i\infty}. \tag{26}$$

Here m_1, m_2 *are the positive lower and upper bounding constants for the norm of* $M(q, \dot{q})$ *matrix,* ϵ*,* K *were defined in (20) and (19), and* $\lambda_{\min}\{\cdot\}, \lambda_{\max}\{\cdot\}$ *are used to denote the minimum and maximum eigenvalues of a matrix, respectively.*

Proof 1: To prove *Theorem 1*, we start by defining the following non-negative scalar function

$$V = \frac{1}{2}r^T M r + \frac{1}{2}e^T e. \tag{27}$$

Using the definitions of m_1, m_2 given in (26), it can be shown that the following bounds hold for (27)

$$\frac{1}{2}\min\{m_1, 1\}\|z\|^2 \leq V(t) \leq \frac{1}{2}\max\{m_2, 1\}\|z\|^2 \tag{28}$$

where $z(t)$ was previously defined. After taking the time derivative of (27), substituting (14) and (24), and applying the skew-symmetry property between the inertia and coriolis matrices, and then cancelling common terms, we have

$$\dot{V} = r^T\left(Y\tilde{\theta} + \xi_r - \frac{r\rho^2}{\|r\|\rho + \varepsilon} - Kr\right) - e^T \alpha e \tag{29}$$

After using (21) we can upper bound the right hand side of (29) as

$$\dot{V} \leq -\min\{\alpha, \lambda_{\min}(K)\}\|z\|^2 + \left[\rho\|r\| - \frac{\|r\|^2\rho^2}{\|r\|\rho + \varepsilon}\right] \tag{30}$$

The bracketed terms of (30) can be manipulated as follows

$$\rho\|r\| - \frac{\rho^2\|r\|^2}{\rho\|r\| + \varepsilon} = \rho\|r\|\left(1 - \frac{\rho\|r\|}{\rho\|r\| + \varepsilon}\right) = \varepsilon\frac{\rho\|r\|}{\rho\|r\| + \varepsilon} \leq \varepsilon, \tag{31}$$

hence using (31), we can further place an upper bound on the right hand side of (30) as shown below

$$\dot{V} \leq -\min\{\alpha, \lambda_{\min}(K)\}\|z\|^2 + \varepsilon \tag{32}$$

From the upper bound on $V(t)$ given in (28), we can further upper bound $\dot{V}(t)$ as follows

$$\dot{V} \leq -\gamma V + \varepsilon \tag{33}$$

where γ was defined in (26). The differential inequality (33) can now be solved to yield [15]

$$V(t) \leq V(0)\exp(-\gamma t) + \frac{\varepsilon}{\gamma}(1 - \exp(-\gamma t)). \tag{34}$$

After applying the bounds of (28) to (34), we obtain the following upper bound for $z(t)$

$$\|z(t)\| \leq \sqrt{\frac{a}{b}\|z(0)\|^2\exp(-\gamma t) + \frac{2\varepsilon}{b\gamma}(1 - \exp(-\gamma t))} \tag{35}$$

where a, b were defined in (26). Based on (35) and the definition of $z(t)$, we can show that the filtered tracking error $r(t)$ and the position tracking error $e(t)$ can be bounded as given by (25) and hence, due to the boundedness of $J^+(q)$ and $J(q)$, we can see from (18) that the bound given in (25) is valid for $e_N(t)$. We can now show that all signals remain bounded by employing standard signal chasing arguments, utilizing assumptions that $x_d(t)$, $\dot{x}_d(t)$, $\ddot{x}_d(t)$, $g(t)$, and $\dot{g}(t)$ are all bounded, and using the fact kinematic and dynamic terms denoted by $M(q)$, $C(q, \dot{q})$, $G(q)$, $J(q)$, and $J^+(q)$ are bounded for all possible $q(t)$. It should be noted that we are not able to show that $q(t)$ remains bounded due to the self-motion of the robot manipulator (*i.e.*, a standard problem associated with redundant manipulators); however, all signals in the manipulator kinematics/dynamics and the control remain bounded independent of the boundedness of $q(t)$ because $q(t)$ only appears as the argument of trigonometric functions. $\boxdot$

IV. MULTIPLE SELF MOTION PERFORMANCE CRITERIA OPTIMIZATION

Up to this point we have shown that, the proposed controller can achieve operational space tracking and still have the redundancy of the robot available to perform sub-tasks. That is the self motion of the redundant manipulator is available to perform at least one sub-task. The question remaining is how to chose and manipulate the performance criterion function $H(q)$ defined in (3) when the robot is subject to multiple performance criteria. An elegant way proposed in [16] suggests that, after the selection of proper performance criteria, an overall performance criteria can be formulated as a weighted sum as follows

$$H(q) = \sum_{i=1}^{s}\omega_i H_i(q) \tag{36}$$

where $H_i(q)$ is the scaler function expressing the ith desired performance criterion, ω_i's are positive valued, scaled functions representing the weight of the corresponding criteria, and $s \in \mathbb{Z}_+$ is the maximum number of self motion (sub-task) criteria. Utilizing (36), the auxiliary joint velocity vector $g(t)$, defined in (3) can be expressed in the following form

$$g(t) = k\sum_{i=1}^{s}\omega_i[\nabla H_i] = k(\omega_1 g_1 + \omega_2 g_3 ... + \omega_s g_s) \tag{37}$$

where self motion control parameter $k \in \mathbb{R}$ was previously defined. Based the formulation given in (37) multiple sub-task prioritization associated to the task can be achieved by adjusting the values of ω_i's subject to the following constraint

$$\sum_{i=1}^{s}\omega_i = P \tag{38}$$

where P is a real valued constant and is used in conjunction with the self motion control parameter k. Taken to the extreme, ω_i can be adjusted through out the execution of the task depending on the importance level of sub-task (e.g. giving more importance to joint limit avoidance than manipulability when the robot is very close to one of its joint limits).

V. SIMULATION RESULTS

To illustrate the performance of the controller proposed in Section 3, we utilized a 3-link revolute, planar robot with the dynamical terms as in [11] except the frictional effects matrix, which is selected as follows:

$$F(\dot{q}) = \begin{bmatrix} f_1(\dot{q}_1) & 0 & 0 \\ 0 & f_2(\dot{q}_2) & 0 \\ 0 & 0 & f_3(\dot{q}_3) \end{bmatrix} \quad (39)$$

Here f_1, f_2, f_3, denote the friction coefficients of each joint constructed explicitly as follows [17]

$$f_i(\dot{q}_i) = f_{di}\dot{q}_i + f_{si}\exp(-f_{\tau i}\dot{q}_i^2)sgn(q_i) \quad (40)$$

where f_{di}, f_{si} and $f_{\tau i}$ denotes the dynamic friction effect, static friction constant and positive constant representing the Stribeck effect. Note that due to the friction model given in (40) robot dynamic model used in the simulations is not linearly parameterizable, therefore adaptive control techniques are not applicable. For simulation purposes, the masses of links 1, 2, and 3 were selected to be 3.60 Kg, 2.60 Kg and 2.00 Kg, respectively. The corresponding link lengths were selected as 0.4m, 0.36m, 0.30m, respectively, and the center of mass of each link was assigned as their midpoints. Based on these selections, the following values were used for the mass and friction parameters of the employed dynamical model in the simulations:

$$\begin{array}{llll} \beta_1 = 1.1956 & \beta_2 = 0.3946 & \beta_3 = 0.0512 & [Kg\,.\,m^2] \\ p_1 = 0.4752 & p_2 = 0.1280 & p_3 = 0.1152 & [Kg\,.\,m^2] \\ f_{d1} = 5.3 & f_{d2} = 2.4 & f_{d3} = 1.1 & [Nm \cdot \text{sec}] \\ f_{s1} = 2.0 & f_{s2} = 1.1 & f_{s3} = 0.6 & [Nm \cdot \text{sec}] \\ f_{\tau 1} = 0.2 & f_{\tau 2} = 0.2 & f_{\tau 3} = 0.2 & [Nm \cdot \text{sec}] \end{array} \quad (41)$$

The constant parameter vector defined in (15) was constructed as follows:

$$\theta = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & p_1 & p_2 & p_3 & f_{d1} & f_{d2} & f_{d3} \end{bmatrix}^T. \quad (42)$$

The manipulator was initialized to be at rest at the following link positions: $q_1 = -0.2rad.$, $q_2 = 1.4rad.$, and $q_3 = 1.9rad.$. The desired task-space trajectory, for all simulations, was selected as follows

$$x_d(t) = \begin{bmatrix} 0.30 + 0.2\cos(t), & 0.40 + 0.1\sin(t) \end{bmatrix}^T. \quad (43)$$

To illustrate the performance of the controller proposed three sets of simulations with different sub-task objectives were performed. In the first simulation, $H(q)$ was set to zero where there was no restriction on the self motion of the robot and only the end-effector tracking objective is enforced. In the second simulation, $H(q)$ was selected, to maximize the manipulability as follows

$$H(q) = \det(JJ^T) \quad (44)$$

where $\det(\cdot)$ denotes the determinant of matrix, the self motion control parameter k and the constant P defined in

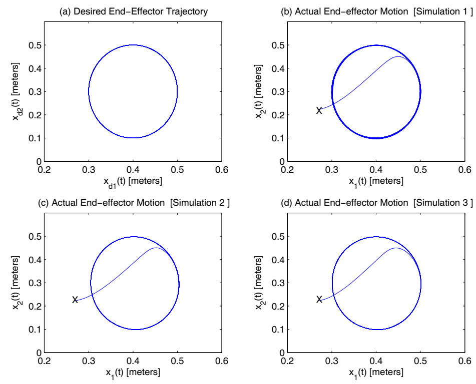

Fig. 1. Desired and Actual Operational Space Trajectories. 'X' marks the Initial Position of the End Effector for each Simulation

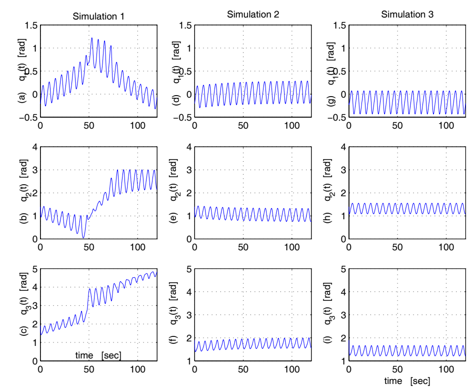

Fig. 2. The Corresponding Link Trajectories for each Simulation

(38) are set to 1. In the third simulation, $H(q)$ was selected as a combination of two different sub-task objectives as

$$H(q) = 0.5\left(\det(JJ^T)\right) + 0.5\left((q_3 - 0.5q_2) - 0.5(q_2 - q_1)\right)^2 \quad (45)$$

where the first term in sub-task function is constructed to maximize the manipulability and the second term attempts to ensure that the optimum link configuration is given by $(q_3 - 0.5q_2) = 0.5(q_2 - q_1)$, (as stated in [6]).

The best results for the controller of (19) were obtained with the following control gains

$$\begin{array}{ll} \alpha = diag\{\; 5 \quad 10 \;\}, & K = diag\{\; 50 \quad 40 \quad 30 \;\}, \\ \rho = 10, & \varepsilon = 0.01 \end{array} \quad (46)$$

with the best guess estimates for the unknown parameters were set to half of their actual values. Figure 1 shows the desired and actual task-space trajectories of each simulation; Figure 2 shows the link trajectories; Figure 3 shows the change of the manipulability measure during the simulation; and Figure 4 shows the control input torque calculated for each simulation.

A. *Discussion on the Simulation Results*

As seen from Figure 1, the end effector position tracking error is below 1cm per axis after the transient. Therefore, it is valid to state that the main objective of the proposed controller, end effector position tracking is achieved regardless of the sub-task. The importance and the effect of the sub-task on the system can be observed from the link position trajectory

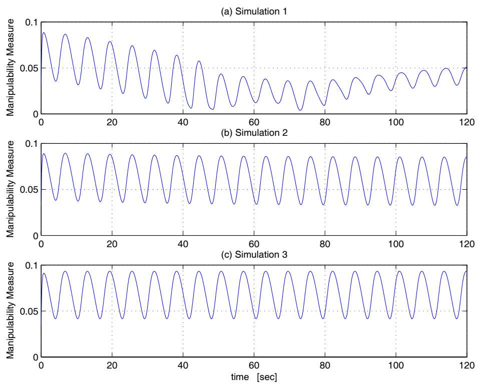

Fig. 3. The Change of the Manipulability Measure for each Simulation

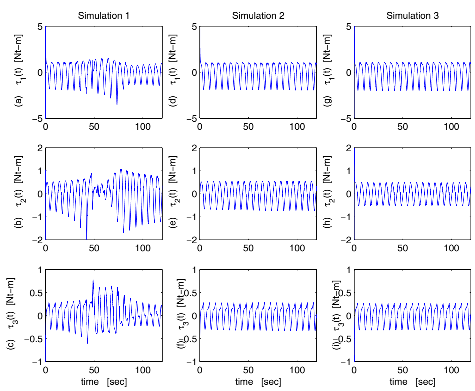

Fig. 4. Control Torque Input Signals for each Simulation

plots given in Figure 2, and from the manipulability measure given in Figure 3 . As can be observed from (a), (b) and (c) sub figures of Figure 2, for the case of $H(q) = 0$, the uncontrolled self motion of the robot causes the link position trajectory variables to drift, and hence, cause the manipulability measure of the robot to change outrageously. Since there were no joint angle limitations enforced in the simulations this drifting phenomena continues however on an actual robot the drifting phenomena would definitely cause the links hits their mechanical limit. In contrast, when the sub-task is selected according to (44), the drift in the link position variables are still there but are not as much when compared to simulation 1, moreover, the corresponding manipulability measure, shown in Figure 3, slowly stabilizes around 60 sec. which indicates that the maximum possible manipulability value is also achieved around the same instant of time. When the sub-task is selected according to (45), no drift on the link trajectories or in the corresponding manipulability measure were observed (see Figure 2 (g), (h) and (i)). When the sub-task is selected according to (45), the controller also keeps the link variables and manipulability measure from drifting too. The manipulability measure of Simulation 3 is very close to that of Simulation 2 (see Figure 3), which may indicate that both manipulability and optimum configuration sub-tasks are achieved simultaneously.

VI. CONCLUSIONS

In this paper, we designed a nonlinear robust controller which achieves globally uniformly ultimately bounded end-effector position and sub-task tracking. By the use of a novel filtered tracking error like term, the proposed controller ensures the regulation of both end-effector position and sub-task tracking inside an ultimate bound in finite time. The applied control strategy uses the pseudo-inverse of the manipulator Jacobian and does not require the computation of the inverse kinematics. Since our control does not place any restriction on the self-motion of the manipulator, extra degrees of freedom are available for sub-tasks like maintaining manipulability, avoidance of mechanical limits and obstacle avoidance. We also showed how to formulate and use the performance optimization criteria when the robot is subject to multiple sub-task tracking objectives. Detailed Simulation results were presented to illustrate the effectiveness of the developed controller.

REFERENCES

[1] Y. Nakamura, "*Advanced Robotics Redundancy and Optimization*", Addison-Wesley Pub. Co., Inc., 1991.
[2] D. N. Nenchev, "Redundancy Resolution through Local Optimization: A Review", *Journal of Robotic Systems*, Vol. 6, pp. 769-798, 1989.
[3] B. Siciliano, "Kinematic Control of Redundant Robot Manipulators: A Tutorial" , *Journal of Intelligent and Robotic Systems*, Vol. 3, pp. 201-212, 1990.
[4] O. Khatib, "Dynamic Control of Manipulators in Operational Space", *6th IFTOMM Congress on Theory of Machines and Mechanisms*, pp. 1-10, New Delhi, Dec. 1983.
[5] H. Seraji, "Configuration Control of Redundant Manipulators: Theory and Implementation", *IEEE Trans. on Robotics and Automation*, Vol. 5, No. 4, pp. 472-490, August 1989.
[6] P. Hsu, J. Hauser, and S. Sastry, "Dynamic Control of Redundant Manipulators", *Journal of Robotic Systems*, Vol. 6, pp. 133-148, 1989.
[7] R. Colbaugh, and K. Glass, "Robust Adaptive Control of Redundant Manipulators", *Journal of Intelligent and Robotic Systems*, Vol. 14, pp. 68-88, 1995.
[8] Z. X. Peng and N. Adachi, "Compliant Motion Control of Kinematically Redundant Manipulators", *IEEE Trans. on Robotics and Automation*, Vol. 9, No. 6, pp. 831-837, Dec. 1993.
[9] Y. Oh and W. K. Chung, "Disturbance-Observer-Based Motion Control of Redundant Manipulators Using Inertially Decoupled Dynamics", *IEEE/ASME Trans. on Mechatronics*, Vol. 4, No. 2, pp. 133-146, June 1999.
[10] E. Zergeroglu, D. M. Dawson, I. W. Walker, and P. Setlur, "Nonlinear Tracking Control of Kinematically Redundant Robot Manipulators", *IEEE/ASME Transactions on Mechatronics*, Vol. 9, No 1, pp 129-132, March 2004
[11] M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, New York: John Wiley and Sons, Inc., 1989.
[12] C. R. Rao, and S. K. Mitra, "Generalized Inverse of Matrices and Its Applications", New York: Wiley, 1971.
[13] T. Yoshikawa, "Analysis and Control of Robot Manipulators with Redundancy", in *Robotics Research- The First International Symposium*, MIT Press, Cambridge, MA, pp. 735-747, 1984.
[14] F. L. Lewis, C. T. Abdallah, and D. M. Dawson, *Control of Robot Manipulators*, New York: MacMillan Publishing Co., 1993
[15] D.M. Dawson, M.M. Bridges, and Z. Qu, *Nonlinear Control of Robotic Manipulators for Environmental Waste and Restoration*, Englewood Cliffs, NJ: Prentice Hall, 1995.
[16] K. Clear, D. Tesar "Incorporating Multiple Criteria in the Operation of Redundant Manipulators," *Proceedings of the IEEE Int. Conf. on Robotics and Automation*, Cincinnati, Ohio, May 1990, pp.618-624.
[17] B. Armstrong-Hélouvry, *Control of Machinery with Friction*, Klawer Academic Publisher, Boston, MA, 1991.